Dissipative process under a boundary perturbation

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A dissipative process in systems subjected to a boundary perturbation is analyzed on the basis of quantum mechanics. We show that the response of the system to the perturbation can be expressed in terms of the first-passage time defined appropriately by quantum mechanics. In other words, the first-passage-time distribution plays the role of the response function in the linear response theory. We apply this formalism to the one-dimensional Anderson model in which a current is introduced at one end of the system and the other is connected to an absorbing wall. We find that the frequency-dependent oscillations of the susceptibility reflect the narrowness of the first-passage-time distribution in disordered systems.

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I. INTRODUCTION

Disordered systems such as amorphous alloys and mixtures of semiconductor particles exhibit interesting properties with regard to transport phenomena because of their disorder. In the last decade, boundary perturbation experiments conducted by using intensity-modulated photocurrent spectroscopy (IMPS) [1] and the frequency response method [2] have been introduced for obtaining information on the transport properties of disordered systems. In ordinary experiments, the perturbation is applied to the entire system; however, in boundary perturbation experiments, a time-dependent perturbation is applied at one end of the system and the response to this perturbation is measured at the other end. The time dependence of the response to the boundary perturbation provides considerable information on the dynamics of the system.

For theoretically analyzing boundary perturbation experiments, Kawasaki *et al.* introduced a random walk model with an absorbing boundary [3,4]. In their study, the *first-passage-time distribution* (FPTD) of a walker with respect to the absorbing boundary was shown to correspond to the response function of the boundary perturbation. Although their analysis was limited to the classical random walk system, it is expected that the correspondence between the first-passage time and the response to the boundary perturbation is universal since the response is generated after the information on the perturbation that is applied at one end propagates to the other end.

In the linear response theory, the response function, which relates the perturbation to the response, is equal to the time correlation function of the fluctuation in the equilibrium state. This random walk model exploited by Kawasaki *et al.* is based on the generalized random walk theory of Odagaki and Lax [5], which related the admittance for localized physical quantities to the random walk on the basis of the linear response theory [6,7]. Therefore, it is important to clarify whether the response function is represented by the FPTD in boundary perturbation experiments for nonlocalized physical quantities subjected to quantum mechanics.

In contrast to classical mechanics, the trajectory of a particle is uncertain in quantum mechanics; therefore, the first-passage time or arrival time is difficult to define. The arrival time in quantum systems has been widely studied [8], and several definitions have been proposed [8–18]. Muga and co-workers proposed a definition based on the complex absorbing potential at the arrival point [9–11], and Marchewka and Schuss defined the arrival time by using Feynman integrals [13–15]. Yamada and Takagi discussed the definition of arrival times within the consistent-histories approach [18]. In contrast to the difficulties in the theoretical treatment of the arrival time, many experiments actually measure the arrival time of elementary particles and atoms using some methods. In these experiments, an apparatus dependence is allowed and the arrival time is clearly defined by the detection of particles. By exploiting this apparatus-dependent arrival time, we show that the first-passage time can be related to the response to the boundary perturbation.

In Sec. II, we formulate the boundary perturbation experiment for quantum systems and prove that the response to boundary perturbation can be expressed by mean of the FPTD. In Sec III, we apply this formulation to a one-dimensional disordered system and show that certain aspects of the response to the boundary perturbation represent the characteristics of randomness. Finally, a summary and discussion are provided in Sec. IV.

II. MODEL FOR BOUNDARY PERTURBATION EXPERIMENT IN A QUANTUM MECHANICAL SYSTEM

A. First-passage-time distribution

We consider particles described by the Schrödinger equation for the state vector $|\psi(t)\rangle$:

$$i\hbar \frac{\partial}{\partial t}|\psi(t)\rangle = \hat{\mathcal{H}}|\psi(t)\rangle, \tag{1}$$

where the Hamiltonian comprises the Hermitian part $\hat{\mathcal{H}}_0$ and non-Hermitian part $\hat{\mathcal{A}}$, which represents the absorbing effect—i.e.,

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$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{A}}. \tag{2}$$

Considering the effects of statistical mixing, we introduce the density operator $\hat{\rho}(t)$ of this system as

$$\hat{\rho}(t) = \sum_m |\psi_m(t)\rangle \rho_m \langle \psi_m(t)|, \tag{3}$$

where $|\psi_m\rangle$ represents the pure state of the system. The time evolution of the density operator follows the Neumann equation

$$\frac{\partial}{\partial t}\hat{\rho}(t) = \frac{1}{i\hbar}[\hat{\mathcal{H}}\hat{\rho}(t) - \hat{\rho}(t)\hat{\mathcal{H}}^\dagger], \tag{4}$$

where $\hat{\mathcal{H}}^\dagger$ represents the Hermite conjugate of Hamiltonian $\mathcal{H}$. Note that $\hat{\mathcal{H}}^\dagger \neq \hat{\mathcal{H}}$ in our system because of the non-Hermitian part $\hat{\mathcal{A}}$. Because of this non-Hermitian part, the trace of the density operator is not conserved and varies with time. The trace of Eq. (4) evolves as

$$\frac{\partial}{\partial t}\mathrm{Tr}\hat{\rho}(t) = \frac{1}{i\hbar}\{\mathrm{Tr}[\hat{\mathcal{H}}\hat{\rho}(t)] - \mathrm{Tr}[\hat{\rho}(t)\hat{\mathcal{H}}^\dagger]\}$$

$$= \frac{1}{i\hbar}\mathrm{Tr}[(\hat{\mathcal{A}} - \hat{\mathcal{A}}^\dagger)\hat{\rho}(t)]. \tag{5}$$

Note that if $\hat{\mathcal{A}}$ represents the absorbing channel, the right-hand side of Eq. (5) would be nonpositive.

Now, we represent the density operator with the basis vectors in space coordinates $|\boldsymbol{r}\rangle$; this is useful when considering boundary perturbation experiments:

$$\rho(\boldsymbol{r},\boldsymbol{r}',t) = \langle \boldsymbol{r}|\hat{\rho}(t)|\boldsymbol{r}'\rangle = \sum_m \rho_m \phi_m(\boldsymbol{r},t)\phi_m^*(\boldsymbol{r}',t). \tag{6}$$

In the above equation, $\phi_m(\boldsymbol{r},t)$ denotes the wave function of the pure state m,

$$\phi_m(\boldsymbol{r},t) = \langle \boldsymbol{r}|\psi_m(t)\rangle, \tag{7}$$

and * represents a complex conjugate. The diagonal elements of the density matrix $\rho(\boldsymbol{r},\boldsymbol{r},t)$ correspond to the probability density of the position of the particle at time t.

The time evolution of $\rho(\boldsymbol{r},\boldsymbol{r}',t)$ obeys

$$\frac{\partial}{\partial t}\rho(\boldsymbol{r},\boldsymbol{r}',t)$$

$$= \left\langle \boldsymbol{r}\left|\frac{\partial}{\partial t}\hat{\rho}(t)\right|\boldsymbol{r}'\right\rangle$$

$$= \frac{1}{i\hbar}\left[\int d\boldsymbol{r}''\langle \boldsymbol{r}|\hat{\mathcal{H}}|\boldsymbol{r}''\rangle\langle \boldsymbol{r}''|\hat{\rho}|\boldsymbol{r}'\rangle - \int d\boldsymbol{r}''\langle \boldsymbol{r}|\hat{\rho}|\boldsymbol{r}''\rangle\langle \boldsymbol{r}''|\hat{\mathcal{H}}^\dagger|\boldsymbol{r}'\rangle\right]. \tag{8}$$

We define matrices $\rho(t)$ and H whose elements are $\rho(\boldsymbol{r},\boldsymbol{r}')$ and $\langle \boldsymbol{r}|\hat{\mathcal{H}}|\boldsymbol{r}'\rangle$, respectively. If we represent the integrals in Eq. (8) as the products of two matrices, Eq. (8) can be simply written as

$$\frac{\partial}{\partial t}\rho(t) = \frac{1}{i\hbar}[H\rho(t) - \rho(t)H^\dagger]. \tag{9}$$

Now, we consider a situation where at time t_0, the initial state is represented by the density matrix ρ_0 and that $G(\boldsymbol{r},\boldsymbol{r}',t;\rho_0,t_0)$ is the solution to Eq. (9). We denote a matrix with elements $G(\boldsymbol{r},\boldsymbol{r}',t;\rho_0,t_0)$ as $G(t;\rho_0,t_0)$. In the system under consideration, the particle flows out to some other system because of the absorbing channel $\hat{\mathcal{A}}$. From a classical viewpoint, this absorption arises when a particle arrives at the absorbing boundary. We can then recognize the decay of the particle density as the arrival to the absorbing boundary. We extend this classical view to our quantum system and define the arrival time distribution up to the absorbing boundary by the decay rate of the particle density. Then, the probability density that the particle arrives at the absorbing boundary $\mathcal{A}$ for the first time at time t when it is in state ρ_0 at time t_0, $F(\mathcal{A},t;\rho_0,t_0)$, is defined by

$$F(\mathcal{A},t;\rho_0,t_0) = -\frac{\partial}{\partial t}\mathrm{Tr}G(t;\rho_0,t_0). \tag{10}$$

On the right-hand side of Eq. (10), the trace of matrix $G(t;\rho_0,t_0)$ is defined as the integral of the diagonal elements in space coordinates:

$$\mathrm{Tr}\, G(t;\rho_0,t_0) = \int d\boldsymbol{r}G(\boldsymbol{r},\boldsymbol{r},t;\rho_0,t_0). \tag{11}$$

By using definition (10), we extend the classical FPTD concept to a quantum system.

B. Boundary perturbation

Now, we determine the relation between this FPTD and the response to the boundary perturbation. Suppose there is an influx of particles into this system; this is represented by an additional term in Eq. (9) as

$$\frac{\partial}{\partial t}\rho(t) = \frac{1}{i\hbar}[H\rho(t) - \rho(t)H^\dagger] + S(t). \tag{12}$$

For the sake of simplicity, we consider a situation where the nondiagonal elements of $S(t)$ are equal to zero; therefore, the particles flow into the system from a definite position. In this situation, the outward flow of particles equals the difference between the decay rate of the density in the system and the influx rate. Then, the outward flow at absorbing boundary $\mathcal{A}$ at time t, $J(\mathcal{A},t)$, is represented as follows:

$$J(\mathcal{A},t) = -\frac{\partial}{\partial t}\mathrm{Tr}\rho(t) + \mathrm{Tr}S(t) = -\frac{1}{i\hbar}\mathrm{Tr}\{H\rho(t) - \rho(t)H^\dagger\}. \tag{13}$$

By using the Green's function $G(t;\rho_0,t)$, the solution to Eq. (12) under the initial condition $\rho(t=0)=\rho_0$ is represented as

$$\rho(\boldsymbol{r},\boldsymbol{r}',t) = G(\boldsymbol{r},\boldsymbol{r}',t;\rho_0,t_0) + \int d\boldsymbol{r}_0\int_{t_0}^{t} dt' G(\boldsymbol{r},\boldsymbol{r}',t;\rho_{\boldsymbol{r}_0},t')S(\boldsymbol{r}_0,t'), \tag{14}$$

where $\rho_{\boldsymbol{r}_0}$ is a matrix whose elements $\rho(\boldsymbol{r},\boldsymbol{r}')$ are represented by

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \begin{cases} \delta(\boldsymbol{r}-\boldsymbol{r}_0) & (\boldsymbol{r}=\boldsymbol{r}'), \\ 0 & (\boldsymbol{r}\neq\boldsymbol{r}'). \end{cases} \tag{15}$$

Substituting Eq. (14) in to Eq. (13), we obtain the final results

$$\begin{aligned} J(\mathcal{A},t) &= -\frac{\partial}{\partial t}\mathrm{Tr}[G(t;\rho_0,t_0)] \\ &\quad - \int d\boldsymbol{r}_0 \int_{t_0}^{t} dt' \frac{\partial}{\partial t}\mathrm{Tr}[G(t;\rho_{r_0},t')]S(\boldsymbol{r}_0,t') \\ &\quad - \int d\boldsymbol{r}_0 \mathrm{Tr}[G(t;\rho_{r_0},t)]S(\boldsymbol{r}_0,t) + \mathrm{Tr}\, S(t) \\ &= F(\mathcal{A},t;\rho_0,t_0) + \int d\boldsymbol{r}_0 \int_{t_0}^{t} dt' F(\mathcal{A},t;\rho_{r_0},t')S(\boldsymbol{r}_0,t'). \end{aligned} \tag{16}$$

In order to derive the second equation in Eq. (16), we used the definition of the FPTD (10) and that of the Green's function—that is, $G(t;\rho_{r_0},t)=\rho_{r_0}$.

Equation (16) implies that $F(\mathcal{A},t;\rho,t_0)$ is the response function of the outward flux, which responds to the perturbation influx $S(t)$. Under the limit that t_0 is infinitely backward, Eq. (16) is reduced to

$$J(\mathcal{A},t) = \int_{-\infty}^{t} dt' \int d\boldsymbol{r}_0 F(\mathcal{A},t;\rho_{r_0},t')S(\boldsymbol{r}_0,t'). \tag{17}$$

Equation (17) represents the linear relation between the boundary perturbation $S(\boldsymbol{r}_0,t)$ and response $J(\mathcal{A},t)$. According to the linear response theory [6,7], the response function $R(t)$ is represented by the response $B(t)$ to the perturbation $A(t)$ as

$$B(t) = \int_{-\infty}^{t} R(t-t')A(t')dt', \tag{18}$$

and $R(t)$ is the time correlation function of the equilibrium fluctuations. Equation (17) suggests that the response function represents the FPTD in boundary perturbation experiments.

For a system in which the Hamiltonian $\hat{\mathcal{H}}$ is invariant under time translation, the FPTD is a function of only the time difference between the departure time t' and the arrival time t; therefore, it can be written as $F(\mathcal{A},t-t';\rho_{r_0})$. In such cases, the susceptibility $\chi(\mathcal{A},\omega;\boldsymbol{r}_0)$ is defined as the Fourier-Laplace transformation of the FPTD:

$$\chi(\mathcal{A},\omega;\boldsymbol{r}_0) \equiv \int_0^{\infty} F(\mathcal{A},t;\boldsymbol{r}_0)\exp(i\omega t)dt. \tag{19}$$

When the time dependence of the perturbation $S(\boldsymbol{r}_0,t)$ is represented by a sinusoidal harmonic oscillation—i.e., $S(\boldsymbol{r}_0,t) = S(\boldsymbol{r}_0)+\Delta S(\boldsymbol{r}_0)\cos(\omega t)$—$\chi(\mathcal{A},\omega;\boldsymbol{r}_0)$ directly represents the response:

$$J(\mathcal{A},t) = \int d\boldsymbol{r}_0 S(\boldsymbol{r}_0) + \int d\boldsymbol{r}_0 \Delta S(\boldsymbol{r}_0)\mathrm{Re}[\chi(\mathcal{A},\omega;\boldsymbol{r}_0)e^{i\omega t}], \tag{20}$$

where we assume that the integral of the FPTD is equal to 1 so that all particles arrive at the absorbing boundary after an infinitely long time.

III. APPLICATION TO A DISORDERED CHAIN SYSTEM

A. Model

We consider the Anderson model as an example of the formulation explained in the previous section; this model is a one-dimensional tight-binding Hamiltonian with an absorption potential:

$$\hat{\mathcal{H}} = \sum_{j=1}^{N} |j\rangle\epsilon_j\langle j| + \sum_{j=1}^{N-1} V(|j\rangle\langle j+1| + |j+1\rangle\langle j|) - iA|N\rangle\langle N|. \tag{21}$$

In this Hamiltonian, $|j\rangle$ represents the Wannier function, which is localized at site j and corresponds to the discrete representation of $|\boldsymbol{r}\rangle$ defined in the previous section. The site energy ϵ_j is a random variable with a uniform probability distribution

$$P(\epsilon) = \begin{cases} \dfrac{1}{W} & (0 \leqslant \epsilon \leqslant W), \\ 0 & (\text{otherwise}), \end{cases} \tag{22}$$

where W is a positive real number. The transfer energy between all nearest-neighbor pairs is a real positive constant V.

The last term on the right-hand side of Eq. (21) is the absorbing potential $-iA$, where A is a positive real constant. This term implies that a particle arriving at site N is absorbed by this potential and flows out of the system. The decay rate of the particles is obtained from Eq. (5) as follows:

$$-\frac{\partial}{\partial t}\mathrm{Tr}\,\hat{\rho}(t) = -\frac{1}{i\hbar}\mathrm{Tr}[(\hat{\mathcal{A}}-\hat{\mathcal{A}}^{\dagger})\hat{\rho}(t)] = \frac{2A}{\hbar}\rho(N,N,t). \tag{23}$$

Here, we used the completeness of the Wannier function—i.e., $\Sigma_j|j\rangle\langle j| = 1$—and the relation $\langle j|\hat{\mathcal{A}}|j'\rangle = -iA\,\delta_{j,N}\delta_{j',N}$.

We consider the influx of a particle to the system at site 1 as a model for the boundary perturbation experiment; therefore, the matrix elements of $S(t)$ in Eq. (12) are $S(i,j,t) = S(t)\,\delta_{i,1}\delta_{j,1}$. As discussed in the previous section, the FPTD from site 1 to the outside of the system is the response function for the outward flux in this system. Since the Hamiltonian (21) is invariant under time translation, the FPTD from site 1 to the outside of the system is a function of the time difference between the departure and arrival. Thus, we write this FPTD as $F(t)$, which is defined by Eq. (10) with $\rho_0(i,j) = \delta_{i,1}\delta_{j,1}$ and $t_0 = 0$. With regard to the boundary perturbation as the particle influx, this FPTD plays a dominant role in the response of the system.

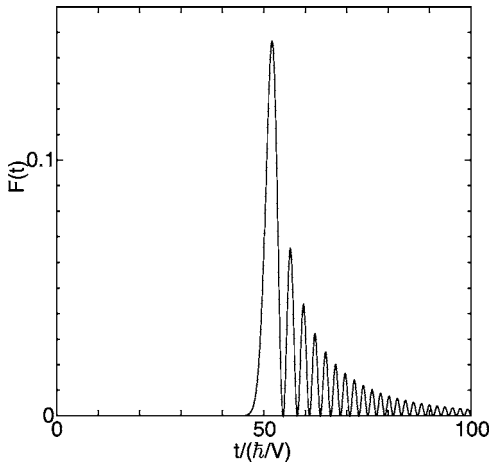

FIG. 1. The first-passage-time distribution for the regular system [$W=0$ in Eq. (21)] as a function of t. The system size is $N=100$, and the intensity of the absorbing potential is set at $A=V$. Although the only results for $A=V$ are presented in this paper, the qualitative feature is almost independent of the choice of A.

B. Results and discussion

We calculated the FPTD for the system described with the Hamiltonian (21) for $A=V$ and $N=100$ by numerical diagonalization of the Hamiltonian. Note that this Hamiltonian is not a Hermitian matrix. Therefore, in some cases, it cannot be diagonalized such that its eigenvalues are degenerate. However, since we consider the random distribution of the site energy ϵ_i and employ a numerical method, the Hamiltonian (21) can be diagonalized in most cases.

Figure 1 shows the FPTD for a regular system—i.e., $W=0$. The shape of $F(t)$ exhibits two main characteristics: first, it is almost zero for t less than approximately $47\hbar/V$, and second, it oscillates as a function of time. The former implies that the particle starting from site 1 rarely flows out of the system before $t_c \simeq 47\hbar/V$. The latter characteristic reflects the discrete property of the system. The system has N finite eigenmodes, and they interfere with each other; hence, the probability of the particle flowing out of the system varies with time.

The Fourier-Laplace transformation of this FPTD — the susceptibility $\chi(\omega)$ — is represented in Fig. 2 as a Colo-Cole plot. In contrast to the Debye semicircle, which is frequently used in relaxation processes, $\chi(\omega)$ represented as a Cole-Cole plot has a vortexlike shape around $\chi(\omega)=0$. This shape mainly originates from the fact that $F(t)$ remains infinitesimal for $t<t_c$. This feature implies that the lower limit of the integral on the right-hand side of Eq. (19) is approximately t_c, and roughly speaking, it leads to an oscillating term $\exp(i\omega t_c)$ in $\chi(\omega)$.

This vortex in the susceptibility disappears with the increase in the width of the site-energy distribution. Figure 3 shows the Cole-Cole plot of the susceptibilities averaged for 1000 samples for $W=0.8V$, $1.6V$, $2.0V$. In comparison to a susceptibility of the regular system, $\chi(\omega)$ for $W=0.8V$ exhibits a smaller vortex, and for $W=1.6V$ and $W=2.0V$, the vortex is not clearly visible. The vortex size can be characterized by two quantities: the minimum value of $\mathrm{Re}\chi(\omega)$, R_m,

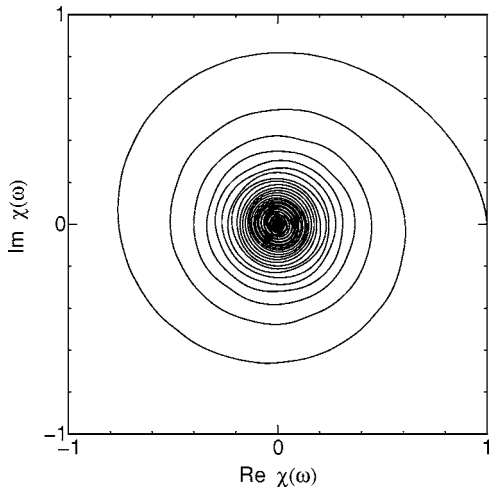

FIG. 2. The Cole-Cole plot of susceptibility $\chi(\omega)$, Eq. (19), of the regular system. The system size is $N=100$, and the intensity of the absorbing potential is set at $A=V$. $\chi(\omega)$ is plotted as the trajectory on the space spanned by $\mathrm{Re}\chi(\omega)$ and $\mathrm{Im}\chi(\omega)$ as the frequency ω is the intervening variable; its range is $10^{-14}<\omega/(V/\hbar)<10^2$.

and the minimum value of $\mathrm{Im}\chi(\omega)$, I_m. In Fig. 4, R_m and I_m are plotted as functions of the width of the site-energy distribution. Both decrease monotonically with an increase in the randomness of the system, W.

The decrease in the vortex size can be attributed to the delay in arrival at the absorbing potential. As the randomness of the site energy increases, several eigenmodes of the system are localized; this causes a delay in the arrival at site N. Therefore, the FPTD is elongated toward the long-time region and the, roughly speaking, absolute value of its Fourier-Laplace transformation rapidly decays as a function of ω. On the other hand, the critical time t_c is almost independent of the randomness and it is determined only by the transfer energy V and system size N. This means that the frequency

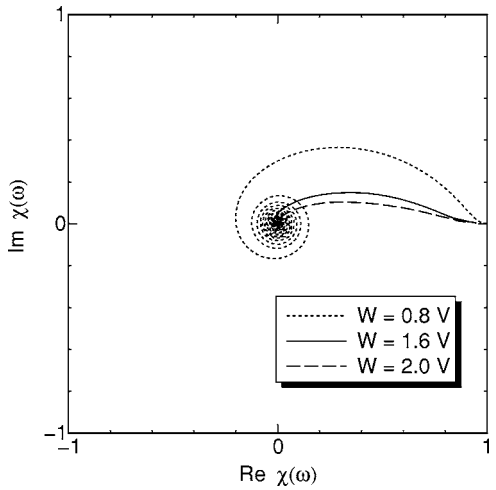

FIG. 3. The Cole-Cole plot of susceptibility of a disordered system. Susceptibilities are averaged for 1000 samples of different site-energy distributions. The range of ω is $10^{-14}<\omega/(V/\hbar)<10^2$. The system size is $N=100$, and the intensity of the absorbing potential is set at $A=V$.

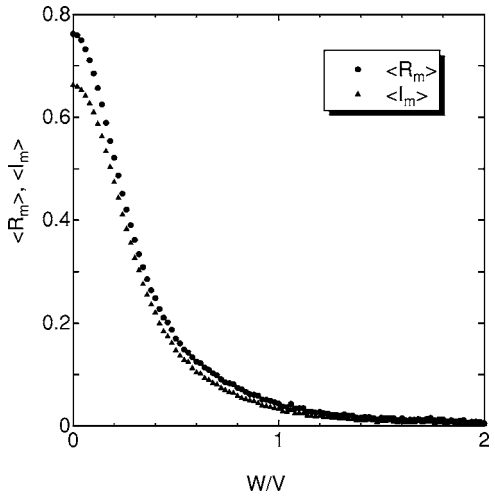

FIG. 4. R_m and I_m averaged for 1000 samples are plotted as a function of W. The system size is $N=100$, and the intensity of absorbing potential is set at $A=V$.

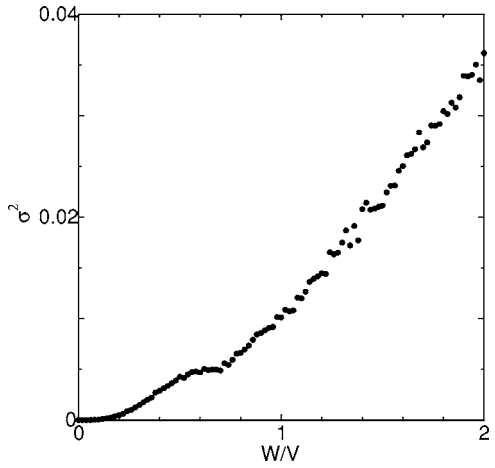

FIG. 6. σ^2 defined in Eq. (24) is plotted as a function of W. The ensemble average was calculated by using 1000 samples. The system size is $N=100$, and the intensity of the absorbing potential is $A=V$.

of oscillation in $\chi(\omega)$ is constant, but the intensity decays very rapidly as the randomness increases. Hence, as a result of the competition between t_c and the decay time τ of the FPTD, the vortex in the Cole-Cole plot vanishes for $\tau \gg t_c$.

Now, we examine each sample in detail. Figure 5 shows the susceptibilities of some sample systems and the averaged susceptibility as a Cole-Cole plot. Note that the susceptibility of each sample as well as the averaged susceptibility has a small vortex around $\chi(\omega)=0$. Therefore, the disappearance of the vortex is not the result of averaging of the susceptibilities that have a large and phase-shifted vortex, but the characteristic of the disordered system. On the other hand, although the averaged susceptibility has only one peak, some susceptibilities in Fig. 5 show more than one peak. The existence of a number of peaks indicates that the disordered system has several modes for arriving at the absorbing potential; the time scales of arrival in these modes are widely different. This characteristic also appears in classical two-dimensional disordered systems [4]; this similarity between quantum one-dimensional systems and classical two-dimensional systems is an open problem in the boundary perturbation experiment for disordered systems.

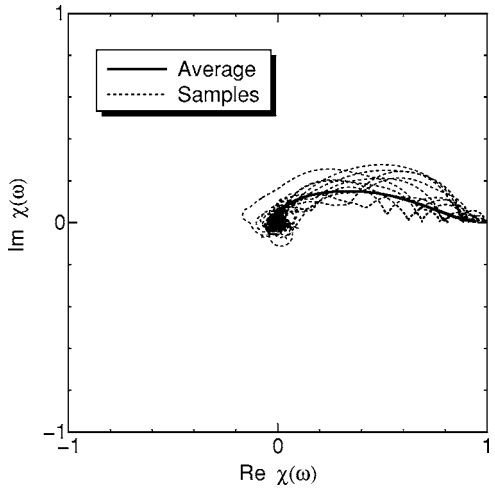

FIG. 5. Susceptibility of various samples for $W=1.6V$ is plotted in a Cole-Cole plot. Dotted lines show susceptibilities of each of the 20 samples, and the solid line shows the susceptibility averaged for 1000 samples. The range of ω is $10^{-14} < \omega/(V/\hbar) < 10^2$. The system size is $N=100$, and the intensity of the absorbing potential is set at $A=V$.

The difference between the sample and averaged susceptibilities increases with an increase in the disorder of the site energy. We employ the maximum variance σ^2 of $\chi(\omega)$ as the indicator of this difference:

$$\sigma^2 \equiv \max_{\omega} \langle |\chi(\omega) - \langle \chi(\omega) \rangle|^2 \rangle. \tag{24}$$

Figure 6 shows σ^2 as a function of the site-energy-distribution width W. An increase in σ^2 implies that for a large W, the response of each sample is different, and this trend becomes more pronounced as W increases. It has been reported that in classical one-dimensional and two-dimensional systems, the susceptibilities to boundary perturbation are non-self-averaging when the disorder is strong; thus, even in a large system, the difference between sample and averaged susceptibilities does not converge to zero [3,4]. Furthermore, it has also been proved that in quantum one-dimensional disordered systems the electric resistance is non-self-averaging [19,20]. Considering the physical similarity between the conductance (inverse of resistance) and first-passage time, it is plausible that in one-dimensional quantum-disordered systems, the susceptibility to boundary perturbation is also non-self-averaging.

IV. DISCUSSION AND SUMMARY

In this paper, a theory for the boundary perturbation experiment in quantum systems was proposed. By using an absorbing potential, we defined the FPTD and showed its relation to the response function. In our definition of the FPTD, the shape of the absorbing potential can be selected.

This implies that we can employ an appropriate absorbing potential to describe experimental situations. Actually the use of some complex absorbing potentials is justified as a model of atomic detection by fluorescence [21–23]. Also, there are many works about the construction of complex absorbing potentials (for a recent review, see, e.g., Ref. [24]).

Since the proposed model is based on the one-particle Schorödinger equation, it is restricted to dilute systems where the interaction between particles is negligible. The extension of this model to interacting systems is a problem to be dealt with in the future.

We also showed the application of the model to one-dimensional disordered systems. The FPTD of the system revealed its response characteristics to the boundary perturbation. While the susceptibility to boundary perturbation oscillates as a function of frequency in the regular system, this oscillation vanishes in a highly disordered system. This results from the delay in the arrival because of site-energy randomness. The disorder in the site energy also causes a discrepancy between the averaged system and each sample. Based on numerical calculations, we expect that the susceptibility to boundary perturbation is non-self-averaging in quantum one-dimensional systems.

In the study of Kawasaki *et al.*, it was proved that in a system where the physical quantities localize, the response function for boundary perturbation experiments is represented by the FPTD. [3,4]. In Sec. II, the same relation was proved for a quantum mechanical system where the physical quantities do not localize. This relation corresponds to that between the time correlation function and the response function in the linear response theory, and we expect that it can be applied to many other systems.

The advantage of the model based on the first-passage time is that various boundary perturbation experiments can be understood in terms of the arrival on the information about the perturbation applied at one side. The first-passage-time model is expected to be applicable not only in cases where information carriers, such as electrons or holes for electric current, clearly exist but also to the transport of quasiparticles.

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